Inter-band B(E2) transition strengths in odd-mass heavy deformed nuclei

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Abstract Inter-band B(E2) transition strengths between different normal parity bands in ^{163}Dy and ^{165}Er are described using the pseudo-SU(3) model. The Hamiltonian includes Nilsson single-particle energies, quadrupole-quadrupole and pairing interactions with fixed, parametrized strengths, and three extra rotor terms used to fine tune the energy spectra. In addition to inter-band transitions, the energy spectra and the ground state intra-band B(E2) strengths are reported. The results show the pseudo-SU(3) shell model to be a powerful microscopic theory for a description of the normal parity sector in heavy deformed odd-A nuclei.

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The pseudo-SU(3) model [1–3] has been used to describe normal parity bands in heavy deformed nuclei. The scheme takes full advantage of the existence of pseudo-spin symmetry, which refers to the fact that single-particle orbitals with $j = l - 1/2$ and $j = (l-2) + 1/2$ in the η shell lie close in energy and can therefore be labeled as pseudo-spin doublets with quantum numbers $\tilde{j} = j$, $\tilde{\eta} = \eta - 1$ and $\tilde{l} = l - 1$. The origin of this symmetry has been traced back to the relativistic Dirac equation [4–6]. In the simplest version of the pseudo-SU(3) model, the intruder level with opposite parity in each major shell is removed from active consideration and pseudo-orbital and pseudo-spin angular momenta are assigned to the remaining single-particle states.

A fully microscopic description of low-energy bands in even-even and odd-A nuclei has been developed using the pseudo-SU(3) model. The first applications used pseudo-SU(3) as a dynamical symmetry, with a single irreducible representation (irrep) of SU(3) describing the yrast band up to the backbending regime [7]. A comparison of the quantum rotor and microscopic SU(3) states [8] provided a classification of the SU(3) irreps in terms of their transformation properties under π rotations in the intrinsic frame [9] and led to the construction of a K^2 operator that plays a crucial role, for example, in determining the excitation energy of the gamma band [10]. On the computational side, the development of a computer code to calculate reduced matrix elements of physical operators between different SU(3) irreps [11] represented a breakthrough in the development of the pseudo-SU(3) model. With this code in place it is possible to include symmetry breaking terms in the interaction, such as pairing which is known to represent important two-body correlations in low-energy configurations. These correlations, while most important for near closed shell nuclei, are also essential for an accurate determination of the low-lying structure of strongly deformed systems.

Once a basic understanding of this overall structure was achieved, a powerful shell-model theory for a description of normal parity states in odd-mass heavy deformed nuclei emerged. For example, the low-energy spectra of several A=159 isotopes [12], as well as their B(E2) intra-band transitions strengths [13], have been successfully described within the pseudo-SU(3) framework using a realistic Hamiltonian.

In the present letter we examine the ability of the pseudo-SU(3) model to describe inter-band B(E2) transition strengths in the odd-A ^{163}Dy and ^{165}Er nuclei. In addition, we examine predictions of the theory for low-energy spectra and ground-state intra-band B(E2) transition strengths in these nuclei. These rare-earth nuclei have one unpaired neutron. This means the normal parity bands have negative parity. The results represent a full implementation of a very ambitious program implied in first applications of the pseudo-SU(3) model to odd-mass nuclei performed nearly thirty years ago [3].

Many-particle states of n_α active nucleons in a given normal parity shell η_α, $\alpha = \nu$ (neutrons) or π (protons), can be classified by the following group chain:

$$\begin{array}{ccccc} \{1^{n^N_\alpha}\} & \{\tilde{f}_\alpha\} & \{f_\alpha\}\ \gamma_\alpha\ (\lambda_\alpha,\mu_\alpha) & \tilde{S}_\alpha\ \ K_\alpha & \\ U(\Omega^N_\alpha) \supset & U(\Omega^N_\alpha/2) \times U(2) \supset & SU(3) \times SU(2) \supset & & \\ & & \tilde{L}_\alpha & J^N_\alpha & \\ & & SO(3) \times SU(2) \supset & SU_J(2), & \end{array} \qquad (1)$$

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where above each group the quantum numbers that characterize its irreps are given and γ_α and K_α are multiplicity labels of the indicated reductions.

The most important configurations are those with highest spatial symmetry [7,14]. This implies that $\tilde{S}_{\pi,\nu} = 0$ or $1/2$, that is, only configurations with pseudo-spin zero for an even number of nucleons or 1/2 for an odd number are taken into account. The basis is built by selecting those proton and neutron SU(3) irreps with the largest value of the second order Casimir operator C_2, and coupling them to a total SU(3) irrep with good angular momentum. Details can be found in previous publications [7,12,13].

The Hamiltonian includes spherical Nilsson single-particle terms for the protons and neutrons ($H_{sp,\pi[\nu]}$), the quadrupole-quadrupole ($\tilde{Q}\cdot\tilde{Q}$) and pairing ($H_{pair,\pi[\nu]}$) interactions, as well as three 'rotor-like' terms which are diagonal in the SU(3) basis:

$$H = H_{sp,\pi} + H_{sp,\nu} - \frac{1}{2}\,\chi\,\tilde{Q}\cdot\tilde{Q} - \; G_\pi\; H_{pair,\pi} \qquad (2)$$
$$- \; G_\nu\; H_{pair,\nu} + \; a\; K_J^2 \; + \; b\; J^2 \; + \; A_{sym}\; \tilde{C}_2.$$

The term proportional to K_J^2 breaks the SU(3) degeneracy of the different K bands [10], the J^2 term represents a small correction to fine tune the moment of inertia, and the last term, $\tilde{C}_2$, is introduced to distinguish between SU(3) irreps with λ and μ both even from the others with one or both odd [9]. This term serves to distinguishes between SU(3) irreps that belong to A-type and B_α-type ($\alpha = 1,2,3$) internal configurations, respectively.

The Nilsson single-particle energies as well as the pairing and quadrupole-quadrupole interaction strengths were taken from systematics [15,16]; only a, b and A_{sym} were used for fitting. Parameter values are listed in Table I and are consistent with those used in the description of neighboring even-even and odd-A nuclei [12,13,17,18].

	χ	G_π	G_ν	a	b	A_{sym}
^{163}Dy	0.00719	0.128	0.104	-0.0400	-0.0040	0.0016
^{165}Er	0.00705	0.127	0.103	-0.0050	-0.0012	0.0011

TABLE I. Parameters used in the Hamiltonian (2).

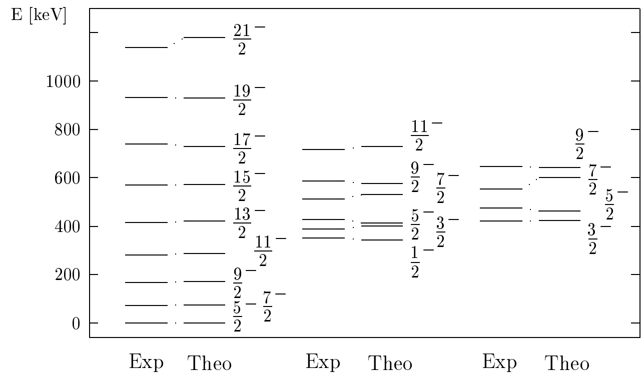

FIG. 1. Energy spectra of ^{163}Dy. 'Exp' represents the experimental results and 'Theo' the calculated ones.

The E2 transition operator is given by [7]

$$Q_\mu = e_\pi Q_\pi + e_\nu Q_\nu \approx e_\pi \frac{\eta_\pi + 1}{\eta_\pi}\tilde{Q}_\pi + e_\nu \frac{\eta_\nu + 1}{\eta_\nu}\tilde{Q}_\nu, \quad (3)$$

with effective charges $e_\pi = 2.3$ and $e_\nu = 1.3$ for both nuclei. These values are very similar to those used in a description of the neighboring even-even [7,17] and odd-A [13] nuclei using the same model; they are larger than those used in standard calculations of B(E2) strengths [15] due to the passive role assigned to nucleons in the unique parity orbitals, whose contribution to the quadrupole moments is parametrized in this way.

Figure 1 shows the calculated and experimental [19] K = 5/2, 1/2, and 3/2 bands for ^{163}Dy. The valence space included 10 protons and 9 neutrons in the normal parity pseudo oscillator shells $\tilde{\eta}_\pi = 3$ and $\tilde{\eta}_\nu = 4$, respectively. The agreement between theory and experiment is in general excellent.

In Figure 2 we present the low-lying energy spectra of ^{165}Er, including the K = 5/2, 3/2, and 1/2 bands built with twelve protons in the normal parity $\tilde{\eta}_\pi = 3$ shell and nine neutrons in the $\tilde{\eta}_\nu = 4$ shell. In this case there is also good agreement between experiment [19] and theory. The model predicts states that have not been seen in the three lowest bands and shows staggering in the excited bands from a simple $J(J+1)$ rule that is expected for a pure rotor model. Due to the lack of experimental information regarding these excited states, it is not possible to confirm the existence of this staggering.

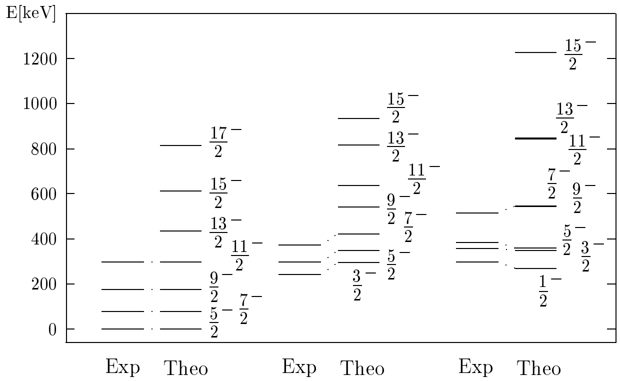

FIG. 2. Energy spectra of ^{165}Er. 'Exp' represents the experimental results and 'Theo' the calculated ones.

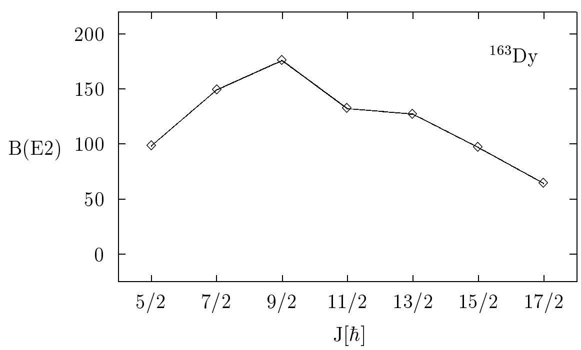

FIG. 3. Theoretical B(E2;$J^- \rightarrow (J+2)^-$) trends in ^{163}Dy [$e^2b^2 \times 10^{-2}$].

In Figure 3 the intra-band B(E2) transition strengths of the ground state band in ^{163}Dy are shown. Experimental values for intra-band B(E2) strengths in ^{163}Dy are reported for only three transitions: 5/2 ($\rightarrow$ 9/2), 97 $\pm$ 18; 9/2 ($\rightarrow$ 13/2), 148 $\pm$ 59; and 11/2 ($\rightarrow$ 15/2), 205 $\pm$ 49 $e^2b^2 \times 10^{-2}$. Theoretical predictions for the 5/2 and 9/2 transitions are in good agreement with experimental values, but the 11/2 prediction falls outside the error bars. For the B(E2;$J^- \rightarrow (J+1)^-$) transition strengths, there is also good agreement for all but the first 5/2 $\rightarrow$ 7/2 transition, which theory overestimates.

The B(E2) values between states belonging the ground state band of ^{165}Er are shown in Figure 4. There are no experimental values reported for these transitions. Inter-band B(E2) transition strengths for both nuclei are shown in Table II.

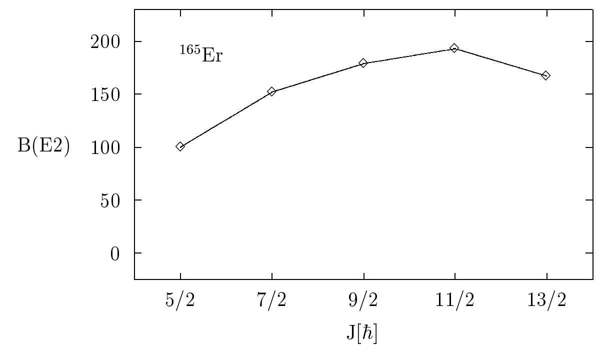

FIG. 4. Theoretical B(E2;$J^- \rightarrow (J+2)^-$) B(E2) trends in ^{165}Er [$e^2b^2 \times 10^{-2}$].

In what may be considered a defining characteristic of these nuclei, inter-band transitions are typically two orders of magnitude smaller than those between states in the same band. This characteristic feature of deformed nuclei is reproduced by the model. A detailed analysis of the wave functions yields a simple explanation. The wave functions of the states belonging to the K = 5/2, 1/2 and 3/2 bands in ^{163}Dy share their main components. Their leading SU(3) component is between 50 and 68% (30,6) and between 24 to 38% (32,2), but the latter enter with a different phases. These phases are responsible for the coherence of B(E2) strengths within each band and at the same time for a cancellation among the terms which leads to small numbers for inter-band transitions.

B(E2) for ^{163}Dy		
$J_i^- \rightarrow J_f^-$	Theo.	Exp. [19]
$1/2_1^- \rightarrow 5/2_1^-$	3.95	4.0 ± 0.8
$3/2_1^- \rightarrow 5/2_1^-$	2.16	1.8 ± 0.6
$3/2_1^- \rightarrow 7/2_1^-$	0.34	3.7 ± 1.6
$5/2_2^- \rightarrow 7/2_1^-$	2.11	3.7 ± 2.1
$5/2_2^- \rightarrow 9/2_1^-$	2.16	3.0 ± 1.6

B(E2) for ^{165}Er		
$J_i^- \rightarrow J_f^-$	Theo.	Exp. [19]
$1/2_2^- \rightarrow 3/2_1^-$	0.26	> 0.54
$1/2_1^- \rightarrow 5/2_1^-$	1.83	1.08 ± 0.13
$1/2_2^- \rightarrow 5/2_1^-$	0.54	> 0.05
$3/2_1^- \rightarrow 7/2_1^-$	1.22	0.53 ± 0.12
$3/2_2^- \rightarrow 7/2_1^-$	1.83	2.09 ± 0.38

TABLE II. Theoretical and experimental inter-band B(E2) transition strengths for ^{163}Dy and ^{165}Er in [$e^2b^2 \times 10^{-2}$].

As shown in Table II, the agreement between experimental [19] and theoretical B(E2) values for five transitions in ^{163}Dy is quite remarkable. For the inter-band B(E2) transition strengths in ^{165}Er, the situation is somewhat different. Specifically, in addition to the relative phase changes noted for the ^{163}Dy case, in ^{165}Er the main SU(3) components of the wave function varies from one band to the next. For example, for the transition $1/2^-_1 \rightarrow 5/2^-_1$, the $1/2^-_1$ state is 97% (32,2) and 2% of (31,4), while the $5/2^-_1$ is comprised of 67 and 31% of these irreps, respectively. So even though both states have the (32,2) irrep as its main component, it enters in different amounts. In addition, the phases of other components are such as to make the B(E2) value very small. For $3/2^-_1 \rightarrow 5/2^-_1$ and $3/2^-_1 \rightarrow 7/2^-_1$ transitions, the SU(3) components of the states involved are nearly the same, ranging from 23 to 31% for the (31,4) irrep and from 67 to 76% for the (32,2) irrep. In these transitions, differences in phases is the main effect yielding the orthogonality of the wave functions, which means small inter-band B(E2) value.

It has been shown that normal parity bands in heavy deformed nuclei can be described quantitatively using the pseudo-SU(3) model. This coupling scheme, which was applied in a previous study to the low-energy spectra [12] and intra-band [13] B(E2) transition strengths in odd-mass heavy deformed nuclei, was used to study inter-band transition strengths. The agreement found with known experimental values is good. This shows that the pseudo-SU(3) scheme is a very creditable theory for achieving a microscopic description of odd-mass deformed rare-earth nuclei. The ability of the model to describe complementary properties of odd-mass heavy deformed nuclei such as g factors, the scissors mode and beta decay transitions will be explored in the future.

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